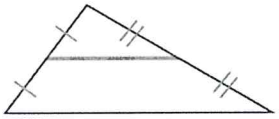
5-1 Midsegments of Triangles

midsegment (noun) MID seg munt

Related Words: midpoint, segment

midsegment



take note

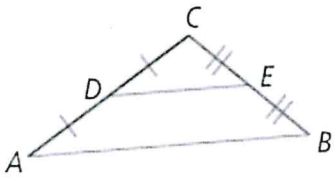
Theorem 5-1 Triangle Midsegment Theorem

Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

If . . .

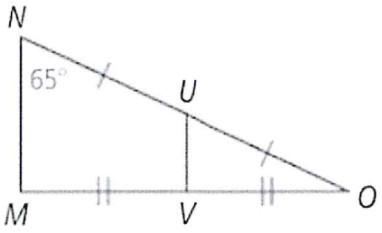
D is the midpoint of $\overline{CA}$ and E is the midpoint of $\overline{CB}$



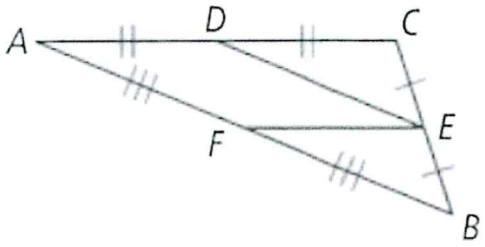
You will prove Theorem 5-1 in Lesson 6-9.

Got It? 1. a. In $\triangle XYZ$, A is the midpoint of $\overline{XY}$, B is the midpoint of $\overline{YZ}$, and C is the midpoint of $\overline{ZX}$. What are the three pairs of parallel segments?

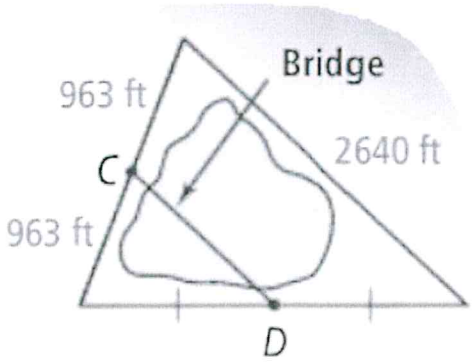
b. **Reasoning** What is $m\angle VUO$ in the figure at the right? Explain your reasoning.



2. In the figure at the right, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?



3. $\overline{CD}$ is a bridge being built over a lake, as shown in the figure at the right. What is the length of the bridge?



5-2 Perpendicular and Angle Bisectors

equidistant (adjective) ee kwih DIS tunt

equidistant

Related Words: equal, distance

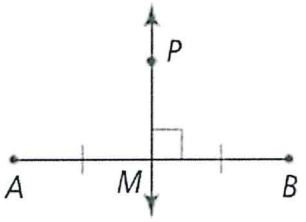
Definition:

take note

Theorem 5-2 Perpendicular Bisector Theorem

Theorem
If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If . . .
$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$

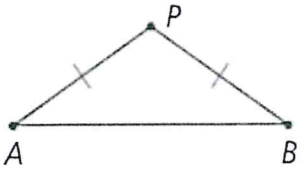


You

Theorem 5-3 Converse of the Perpendicular Bisector Theorem

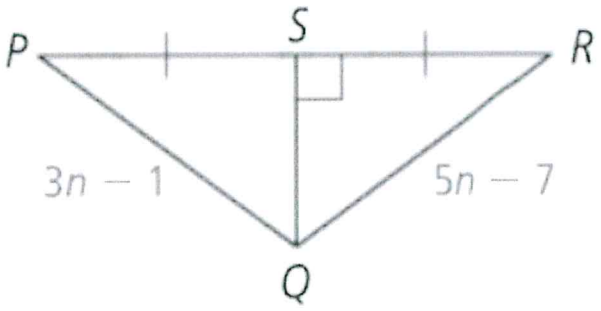
Theorem
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If . . .
$PA = PB$

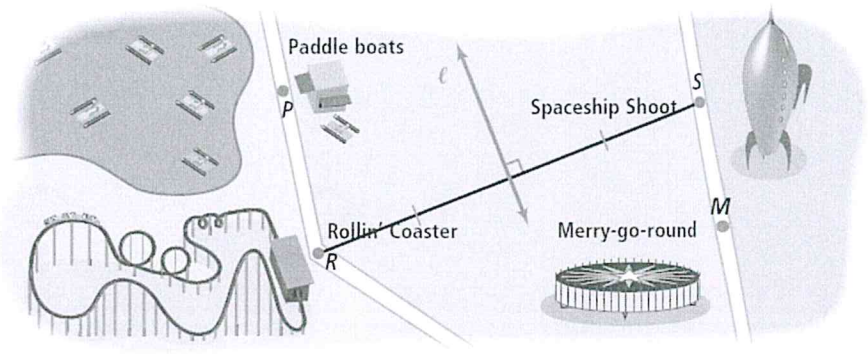


You

1. What is the length of $\overline{QR}$?



Got It? If the director of the park at the right wants a T-shirt stand built at a point equidistant from the Spaceship Shoot and the Rollin' Coaster, by the Perpendicular Bisector Theorem he can place the stand anywhere along line ℓ. Suppose the park director wants the T-shirt stand to be equidistant from the paddle boats and the Spaceship Shoot. What are the possible locations?



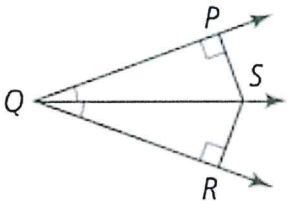
The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This distance is also the length of the shortest segment from the point to the line.

take note

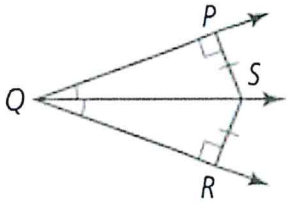
Theorem 5-4 Angle Bisector Theorem

Theorem
If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

If . . .
$\overrightarrow{QS}$ bisects $\angle PQR$, $\overline{SP} \perp \overrightarrow{QP}$, and $\overline{SR} \perp \overrightarrow{QR}$



Then . . .
$SP = SR$

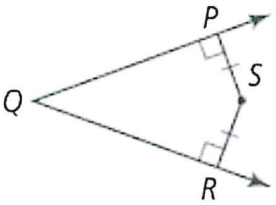


You will prove Theorem 5-4 in Exercise 34.

Theorem 5-5 Converse of the Angle Bisector Theorem

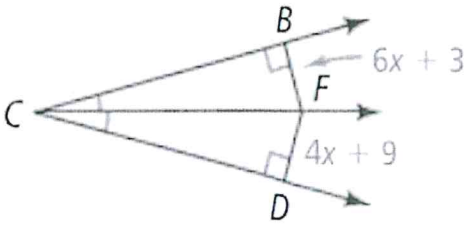
Theorem
If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.

If . . .
$\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$, and $SP = SR$



You

3. What is the length of $\overline{FB}$?



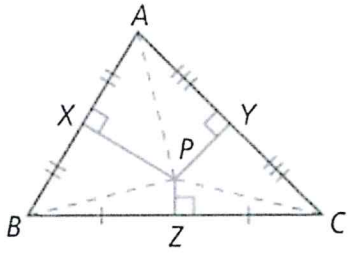
5-3 Bisectors in Triangles

take note

Theorem 5-6 Concurrency of Perpendicular Bisectors Theorem

Theorem
The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

Diagram



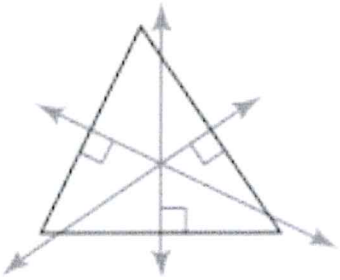
Symbols
Perpendicular bisectors $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are concurrent at P.

$PA = PB = PC$

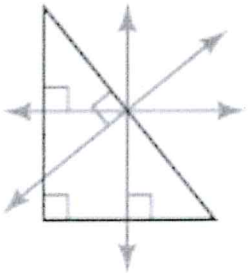
The point of concurrency of the perpendicular bisectors of a triangle is called the

The circumcenter of a triangle can be inside, on, or outside a triangle.

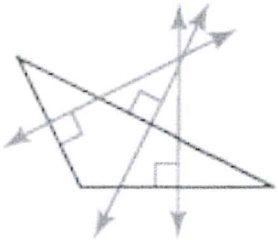
Acute triangle



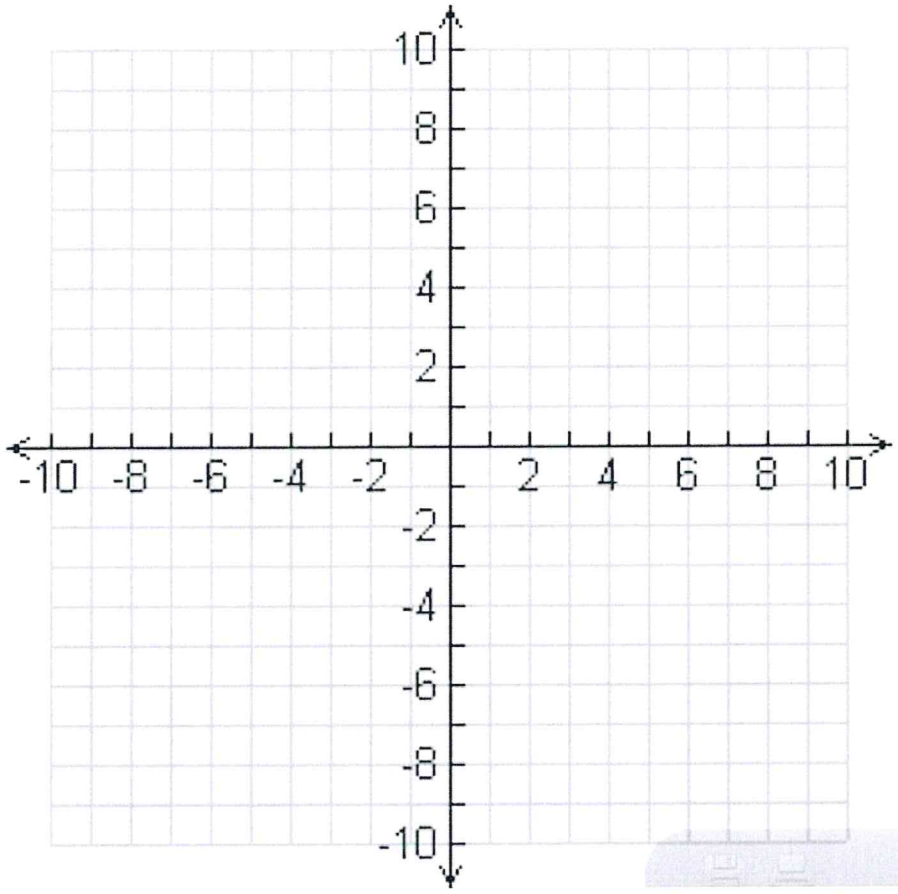
Right triangle



Obtuse triangle



1. What are the coordinates of the circumcenter of the triangle with vertices $A(2, 7)$, $B(10, 7)$, and $C(10, 3)$?



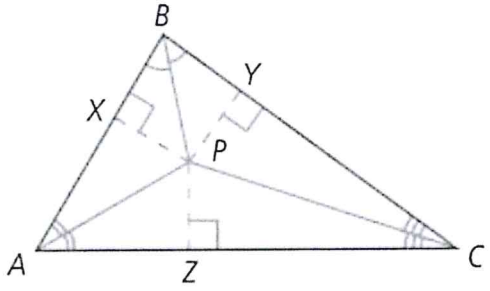
take note

Theorem 5-7 Concurrency of Angle Bisectors Theorem

Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Diagram

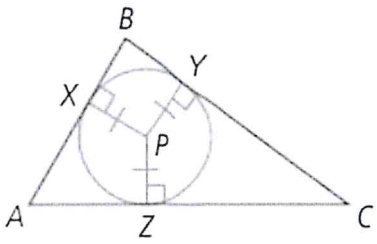


Symbols

Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at P.

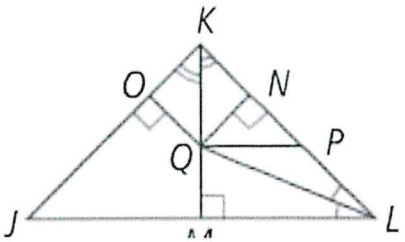
You will prove Theorem 5-7 in Exercise 24.

The point of concurrency of the angle bisectors of a triangle is called the i



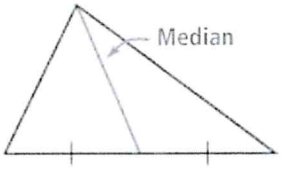
3. **a.** $QN = 5x + 36$ and $QM = 2x + 51$. What is QO?

b. Reasoning Is it possible for QP to equal 50? Explain.



5-4 Medians and Altitudes

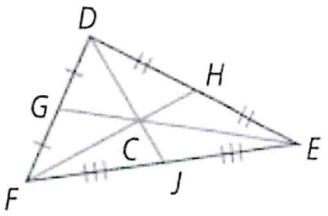
medians. A **median of a triangle** is a segment whose endpoints are a vertex and the midpoint of the opposite side.



take note

Theorem 5-8 Concurrency of Medians Theorem

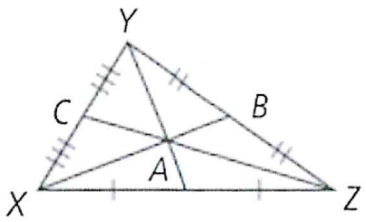
The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.



vill prove Theorem 5-8 in Lesson 6-9.

In a triangle, the point of concurrency of the medians is the

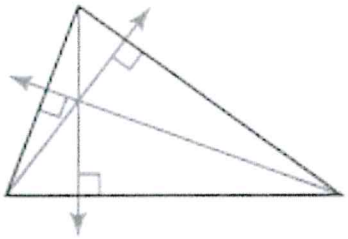
1. a. In the diagram for Problem 1, $ZA = 9$. What is the length of $\overline{ZC}$?
   b. **Reasoning** What is the ratio of ZA to AC? Explain.



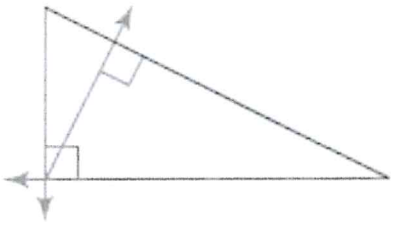
[ ] is the perpendicular segment from a vertex of the triangle to the line containing the opposite side. An altitude of a triangle can be inside or outside the triangle, or it can be a side of the triangle.

The lines that contain the altitudes of a triangle are concurrent at the [ ] **the triangle.** The orthocenter of a triangle can be inside, on, or outside the triangle.

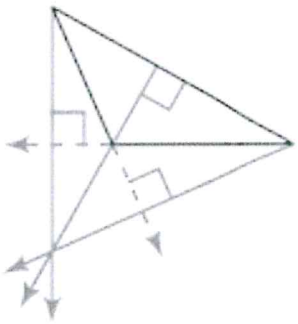
Acute triangle



Right triangle

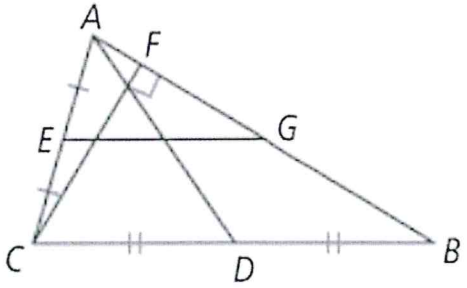


Obtuse triangle

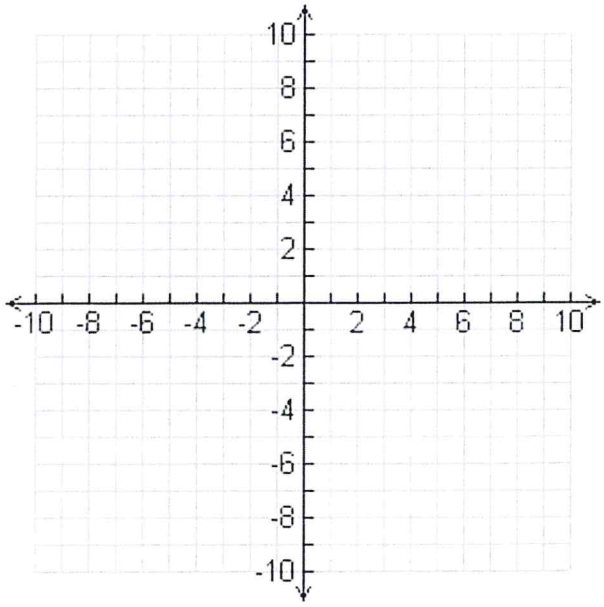


2. For $\triangle ABC$, is each segment a *median*, an *altitude*, or *neither*? Explain.

a. $\overline{AD}$ b. $\overline{EG}$ c. $\overline{CF}$

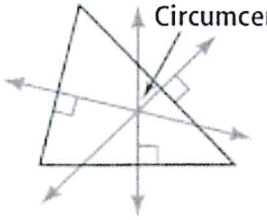
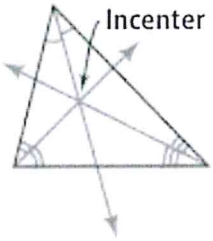
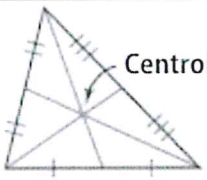
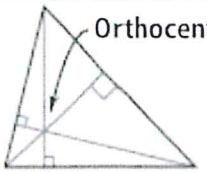


3. $\triangle DEF$ has vertices $D(1, 2)$, $E(1, 6)$, and $F(4, 2)$. What are the coordinates of the orthocenter of $\triangle DEF$?



take note

Concept Summary Special Segments and Lines in Triangles

 Circumcenter	 Incenter	 Centroid	 Orthocenter

5-5 Indirect Proof

indirect (adjective) **in duh** REKT

Definition: **Indirect** means not direct in course or action, taking a roundabout route to get to a point or idea.

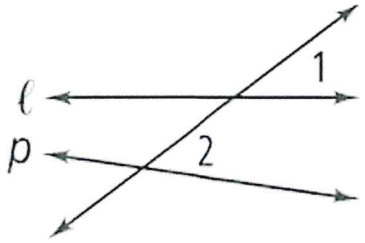
Math Usage: In **indirect** reasoning, all possibilities are considered and then all but one are proved false. The remaining possibility must be true.

Key Concept Writing an Indirect Proof

Think About a Plan Write an indirect proof.

Given: $\angle 1 \ncong \angle 2$

Prove: $\ell \nparallel p$



1. Suppose you want to write an indirect proof of each statement. As the first step of the proof, what would you assume?
   a. $\triangle BOX$ is not acute.
   b. At least one pair of shoes you bought cost more than \$25.

2. a. Which two statements contradict each other?
      I. $\triangle XYZ$ is acute.
      II. $\triangle XYZ$ is scalene.
      III. $\triangle XYZ$ is equiangular.
   b. **Reasoning** Statements I and II below contradict each other. Statement III is the negation of Statement I. Are Statements II and III equivalent? Explain your reasoning.
      I. $\triangle ABC$ is scalene.
      II. $\triangle ABC$ is equilateral.
      III. $\triangle ABC$ is not scalene.

3. **Given:** $7(x + y) = 70$ and $x \neq 4$.
   Prove: $y \neq 6$

5-7 Inequalities in Two Triangles

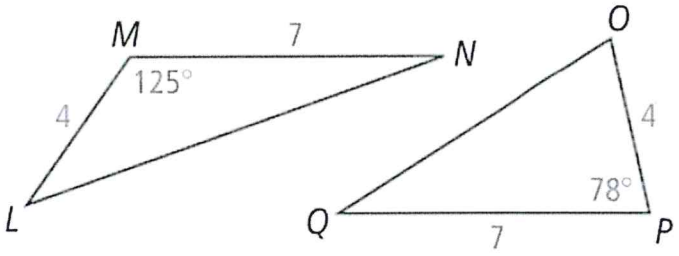
take note

Theorem 5-13 The Hinge Theorem (SAS Inequality Theorem)

Theorem

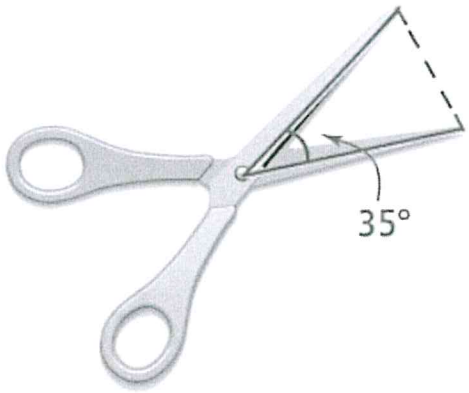
If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

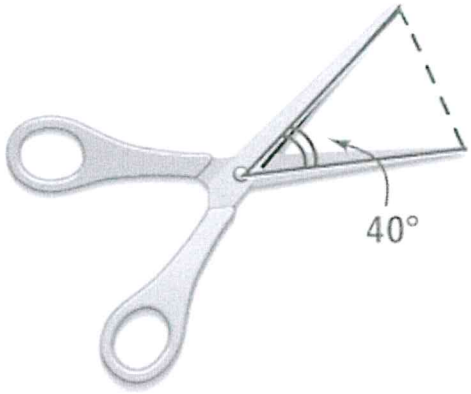
1. a. What inequality relates LN and OQ in the figure at the right?



b. **Reasoning** In $\triangle ABC$, $AB = 3$, $BC = 4$, and $CA = 6$. In $\triangle PQR$, $PQ = 3$, $QR = 5$, and $RP = 6$. How can you use indirect reasoning to explain why $m\angle P > m\angle A$?

2. The diagram below shows a pair of scissors in two different positions. In which position is the distance between the tips of the two blades greater? Use the Hinge Theorem to justify your answer.





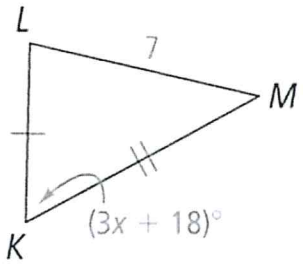
take note

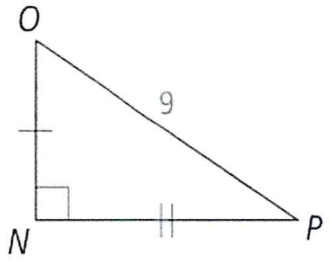
Theorem 5-14 Converse of the Hinge Theorem (SSS Inequality)

Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third sides are not congruent, then the larger included angle is opposite the longer third side.

3. What is the range of possible values for x in the figure at the right?





4. **Given:** $m\angle MON = 80$, O is the midpoint of $\overline{LN}$
Prove: $LM > MN$